Finding Maximum power of a number that divides a factorial number

To find the maximum power of a number which divides a factorial number, we need to consider how many of these numbers contained in the factorial.

Solved Example 1:

The maximum power of 5 in 60!

Sol: 60! = 1 x 2 x 3 ....60 so every fifth number is a multiple of 5. So there must be 60/5 = 12

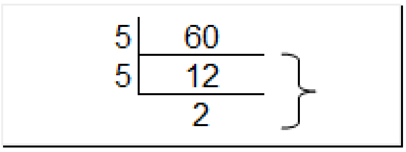
In addition to this 25 and 50 contribute another two 5's. so total number is 12 + 2 = 14

Short cut: $[\frac{60}{5}]+[\frac{60}{5^2}] = 12+2 = 14$

Here [ ] Indicates greatest integer function.

Shortcut:

Divide 60 by 5 and write quotient. Omit any remainders. Again divide the quotient by 5. Omit any remainder. Follow the procedure, till the quotient not divisible further. Add all the numbers below the given number. The result is the answer.



Solved Example 2:

Find the highest power of 12 that divide 49!.

Sol: We should commit to the memory that the above method is applicable only to prime numbers. So we should write 12 in its prime factors. 12 = $2^2 \times 3$

We find the maximum power of 2 in 49! = $\frac{49}{2}+\frac{49}{4}+\frac{49}{8}+\frac{49}{16}+\frac{49}{32}$ = 24 + 12 + 6 + 3 + 1 = 46

So maximum power of 2^2 in 49! is 23.

Now we find the maximum power of 3 in 49! = $\frac{49}{3}+\frac{49}{9}+\frac{49}{27} = 16+5+1 = 22$

$\Rightarrow 49! = (2^2)^{23} \times 3^{22} \times \text{someK}$

So 22 is the maximum power of 12 that divides 49! exactly.

Solved Example 3:

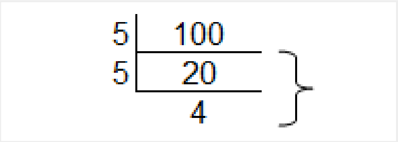
How many zero's are there at the end of 100!

Sol: A zero can be formed by the multiplication of 5 and 2. Since 100! contains more 2's than 5's, we can find the maximum power of 5 contained in 100!

For your understanding:

$\Rightarrow \frac{100}{2} + \frac{100}{4} + \frac{100}{8} + \frac{100}{16} + \frac{100}{32} + \frac{100}{64}$ = 50 + 25 + 12 + 6 + 3 + 1 = 97

$\Rightarrow \frac{100}{5} + \frac{100}{25} = 20 + 4 = 24$



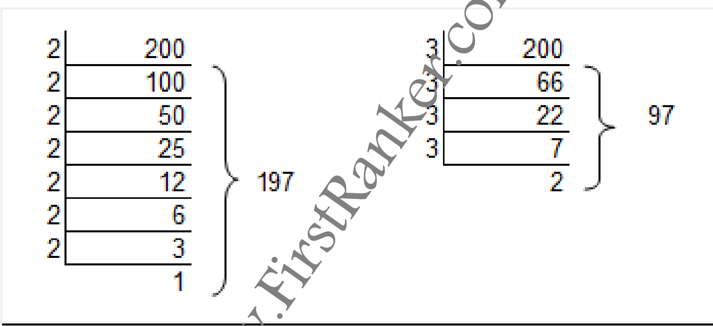
So there are 24 zero's at the end of 100!

Solved Example 4:

By which number the expression $\frac{200!}{12^{100}}$ should be multiplied so that the given expression becomes an integer

Sol: 12^{100} = $4^{100} \times 3^{100}$ = $2^{200} \times 3^{100}$

Now we have to find the maximum power of 2 and 3 in numerator.



$\frac{200!}{12^{200}} = \frac{2^{197} \times 3^{97} \times \ldots}{2^{200} \times 3^{100}}$

To divide the numerator, we need to multiply it with $2^3 \times 3^3 = 216$

Solved Example 5:

What is the maximum power of 3 in the expansion of 1! × 2! × 3! × . . . . × 100!

Sol: Given 1! × 2! × 3! × . . . . × 100!. We rewrite this expression by writing as $1^{100} \times 2^{99} \times 3^{98} \times \ldots \times 100^1$

This is possible as each term contains 1. From 2! on wards each term contains 2. So they are total 99. Similarly, only last term contains 100. So it has power of 1.

Now we have to calculate number of 3's.

$3^{98}, 6^{95}, 9^{92} \ldots, 99^2$

Observer here the power and base sum = 101. So it is easy to form this series.

Total powers of 3 = 98 + 95 + 92 + ... + 2

Number of terms = $\frac{l - a}{d} + 1$ = $\frac{98 - 2}{3} + 1$ = 33

Sum = $\frac{n}{2}(a + l)$ = $\frac{33}{2}(98 + 2)$ = 1650

Also, the powers of 9's, 18's, 27's contains another 3. They contribute more number of 3's.

So $9^{92}, 18^{83}, 27^{74} \ldots, 99^2$

Number of terms = $\frac{1-a}{d}+1$ = $\frac{92-2}{9}+1$ = 11

Sum = $\frac{n}{2}(a+1)$ = $\frac{11}{2}(2+92)$ = 517

Also, the powers of 27's, 54's, 81's contains another 3.

So 27^{74}, 54^{47}, 81^{20}

Their sum = 74 + 47 + 20 = 141

Finally, power of 81 contains another 3. So it contributes another 20.

Total = 1650 + 517 + 141 + 20 = 2328.